1. Renata Russell (USAFA)

Playing with Games: A Variation on Nim

Nim is a two-person game in which the players take turns removing objects from a collection of piles. On their turn, a player must remove at least one object from any one of the piles. The player may remove as many objects as they would like as long as they are all from the same pile. A player wins when they take the last object. We examine a variation of the Game of Nim in which the piles have links between them -- that is, removing enough objects from one pile may automatically remove objects from another pile. To what extent does this change the classification of winning and losing positions?

2. Jo Spain (PPSC)

Sandwich Science: Exploring the Ham Sandwich Theorem

The funnily named Ham Sandwich theorem is more complicated and serious than first thought. It states that n-dimensional space can be bisected by a single dimensional hyperplane. This presentation will explain the theorem and then it's potential applications to real-world problems such as gerrymandering and telecommunications.

3. Zain Nathan (USAFA)

Pushup Problem Generalization

There is a USAFA tradition involving performing pushups at a football game. At the end of the game, the total number of pushups performed is the quantity in question; what can be reached and what can't? In this talk we'll answer this question for two possible pushup scoring values and briefly explore how this research could be continued for the future.

4. Licia Lamb (MSU)

Factorial Frenzy

Factorials have been a topic of fascination and study for millennia, serving as the quiet foundation for a plethora of mathematical concepts. While complicated at first glance, there are distinct and reliable patterns given by factorials. For example, each factorial greater than 1! is an even number. Approaching factorials through this lens of pattern recognition, we observe that for $n \geq 5$, $n!$ has at least one trailing zero. In this presentation, we give a formula that predicts the number of trailing zeros on $n!$ and proceed to further generalize this idea to determine the maximal k for which $n!$ is divisible by m^k. The process of deriving this function and our results highlight the impact of prime factorizations within large numbers. As we explore the continuously expanding world of factorial patterns, we can utilize them to demonstrate the value and applications of theoretical mathematics.

5. Delaney Finley, Alex Le (USAFA)

Hypertemporal Analysis of Light Fixtures

The stroboscopic effect - the change in motion perception due to temporal fluctuations in luminance - is known to cause adverse health effects. This effect is most commonly found in light flicker caused by the electrical grid's alternating current. Due to current rectification, most light fixtures flicker at exactly twice the grid frequency. The hypertemporal performance of event-based sensors (referred to as EBS cameras) allows us to observe this flickering effect. In this project, we apply Fourier analysis to EBS recordings of desk lights in order to analyze phase behavior. In addition, we introduce the phase-spectrogram analysis technique to effectively communicate phase information. Our results show that the frequency and phase of the light flicker can be clearly determined. This suggests that Fourier-based approaches are useful for studying phase behavior of lights and, by extension, the stroboscopic effect.

6. Joey Fischer (DU)

Efficacy of Artificial Intelligence Models in Advanced Mathematics

Considering the rapid expansion of the use cases of AI in education, business, and daily life, this research aims to assess the efficacy of AI chatbots when faced with mathematical problems. The study aims to create a better understanding of the shortcomings of AI in logical problems, and place them in the larger framework of AI development. Five AI chatbots were provided with eight mathematical questions and graded according to the accuracy of their provided solutions. The AI models achieved an average success rate of 62.5%, suggesting a large gap in artificial intelligence's understanding of advanced mathematics. This error likely stems from the lack of mathematical theoretical foundations in AI training datasets and reliance on statistical approximations rather than deductive reasoning. These shortcomings are important to consider when implementing an artificial intelligence into critical applications.

7. Clara V. Bailey (PPSC)

Harmonic Series: The Calculus of Music and All That Jazz

This presentation explores the mathematical reasoning behind harmonies in music as well as the diverse harmonious relationship represented by the harmonic series. This presentation additionally includes the theme of how math is used to model, simplify, and connect real world elements to expand their applications.

8. Carter Baskin, Jack Roe, (Western)

Abelian Sandpiles

Our goal for this project is to explain the idea of the abelian sandpile and its associated operation. This includes an explanation of how to describe a sandpile as an undirected graph with weighted vertices, a mathematical representation of how to add sandpiles together, and what happens when a vertex exceeds its maximum allowed weight. When this happens, a scenario called "toppling" occurs, and we aim to provide a method to streamline this toppling, especially when more than one vertex exceeds its weight. This streamlined version comes from linear algebra, and the construction of a "toppling matrix"

9. Cale Curtis

An Introduction to Time Scales

A time scale is any arbitrary nonempty closed subset of the real numbers. In this talk, we provide an introduction to the time scales, covering the classification of points within a given time scale, the forward and backward jump operators, and the graininess function. Additionally, we explore how one would take a derivative of a function in a time scale, since the usual limit definition does not always apply.

10. Julia Ahrendt (USAFA)

Predicting Recidivism

Recidivism is a critical issue in the criminal justice system with significant societal and economic impacts. This project develops and compares predictive models to identify key factors using methods in R such as logistic regression, decision trees, boosted trees, and random forest.

The findings aim to inform reintegration efforts, with practical applications for Benevolence Farm, a nonprofit supporting formerly incarcerated women. By uncovering actionable insights, this project seeks to enhance program effectiveness and contribute to evidence-based strategies for reducing recidivism.

11. Blake Farmer, Maxyn Hallare (USAFA)

Stabilizing the Unstable: A mathematical analysis of military-insurgent-civilian interactions

By examining examples of past insurgencies such as the Irish Civil War, the Iraq War, and the Philippine Revolutionary movement, we developed an ODE model to analyze the dynamics between militaries, insurgencies, and civilians. Our model tells the story of natural growth, decay, support, and conflict experienced in the system. Using the Jacobian to perform a stability analysis, we identify the necessary conditions for long-term stability, or instability, in conflicted areas, particularly in relation to civilian support. Failure to properly understand counter-insurgency dynamics has resulted in setbacks for military commanders. Our findings offer quantitative insights into the dynamics of a conflicted area and provide a springboard into analyzing how to better deal with counter-insurgency operations in light of the Global War on Terror.

12. Liam Markus (USAFA)

A Multicompartmental Circulatory System Model

The human circulatory system is introduced in regards to its components, basic principles, and important parameters. In addition, the importance of the circulatory system is explained in relation to its value to the medical profession. Four models are then constructed that each increase in relative difficulty. There is the "simple" model followed by the "linear" model. Then the multicompartment model is introduced and a fourth model is constructed that integrates some assumptions that ease computation and complexity. For each model, the diagram is provided along with the equations. For the fourth model, sensitivity analysis is performed using a parameter variation table, which allows for quantitative and qualitative inferences on the effect of parameter changes. This is particularly useful in understanding the physiological impact of diseases such as coronary artery disease (CAD), hypertension (HTN), left-sided heart failure, and right-sided heart failure. However, there are certain diseases that the model does not adequately address due to some of the models' assumptions. To address the multicompartment models' pitfalls, bleeding and cardiomyopathy are discussed along with next steps to improve the model.

13. Giorgio Simoncioni (USAFA)

3N+1: It's Hailing...Exponents?

Abstract: "3N+1: It's Hailing…Exponents?" is a presentation centered around the well-known but little-understood Collatz Conjecture, first reviewing the basics of Collatz, then diving into why the conjecture has confounded analytical approaches for a century. Finally, a novel approach to the problem is reviewed in detail, providing a surprising result regarding the conjecture and leaving the audience wondering whether the conjecture might be nearing an end.

14. Taylor McManus (USAFA)

Introduction to Fully Homomorphic Encryption and its Acceleration

With the rise in cloud computing, Fully Homomorphic Encryption (FHE)—enabling secure data storage and manipulation in untrusted environments—has become increasingly necessary. As it stands, FHE algorithms require a computational cost beyond widely available capacities, preventing real-world use. My capstone reading course explores an existing method of algorithmic acceleration aimed at reducing the cost of computing polynomial products using the Number Theoretic Transforms domain.

15. Lucas Briggs (USAFA)

Proving Without Words that Root Three is Irrational

There are many proofs that the square root of three is irrational. In this talk we use a bijective argument to show this classic result through a new lens. Not only is this an aesthetically satisfying perspective, but it is a beautiful example of how combinatorics can be used to prove results in other areas of mathematics.

16. Aaron Anderson (UCCS)

Quantifying the Magnitude of Chaos in a Driven Pendulum via Lyapunov Exponents

Given the differential equation that models a physical pendulum with a sinusoidal driving force, we determine the Lyapunov exponent under different combinations of driving amplitude and frequency. We then find that any such combination of amplitude and frequency produces a Lyapunov exponent that falls into one of two categories. Our main observation is the distinct boundary between these categories, and how it signals the chaos of the system.

17. Chris Galatos (STEM School)

Symmetries of Platonic Solids

In this talk, we report on the research project that we pursued at the 2024 Yau Mathcamp in Shanghai, China this past summer. We explore the rotational symmetries of the Dodecahedron using group actions and geometry, and the reflectional symmetries using group theory and linear transformations. Then we will complete the classification of all the finite rotational groups in 3D space using Burnside's Lemma and other group actions as well.

18. Lara M. Bailey (PPSCS)

Where Infinity Meets the Finite: Proving Repeating Decimals are Exact Values Using Adics

This presentation will focus on the study of p-adics and 10-adics with the goal of proving that infinitely repeating numbers equal finite values. This presentation will explain how simple repeating digits will equal finite values, both when the digits are on the left and right side of a decimal point. Additionally, the concept of adding, subtracting, and multiplying infinite values will be discussed, as well as how using prime numbers as opposed to base 10 affects these processes.

19. Reese Combs (PPSC)

The Mathematics of the Cardiovascular System

The cardiovascular system is a dynamic and constantly changing system. When it comes to studying the system, math can provide the foundation for a framework that can model these dynamic changes. Poiseuille equations can help measure the amount of blood flow though veins and arteries while Ohms law can help model the changes that the cardiovascular system experiences. By understanding the mathematics of the cardiovascular system, it can help provide a better understanding which ultimately leads to further medical advances and better treatment.

20. Luck Henderson , Enrique Mercado (CSU)

Generating examples of surjective arboreal Galois representations for quadratic polynomials

The goal of this project is to find examples of $f(x)=x^2+c$ and a with a,c in the rational numbers where we can verify that for every n, there is a prime p which divides the numerator of $f^n(0)-a$. Additionally, p cannot divide any earlier numerators, and p must divide the numerator an odd number of times. It has been proven that pairs f,a satisfying these criteria will have surjective arboreal Galois representations, which can be applied to density problems in arithmetic dynamics.

21. Daniela Young (PPSC)

Don't Take 'Nothing' For Granted: A History of Zero

Although many students today take the number zero for granted, it has a surprisingly long and controversial history. This presentation will briefly overview the history of zero, tracing its evolution around the world and exploring some of the reasons it was such a controversial idea. Zero was first used as a placeholder in ancient Babylon. It spread from there, with vastly different results in different cultures. While rejected by mathematicians in Ancient Greece, zero was adopted by those in the Indian subcontinent, where it was first conceptualized as its own number. The journey of zero continued into the Arab world, before finally reaching Europe. This presentation will highlight the factors that made zero so controversial in some cultures, its significance in the evolution of mathematics, and some of its philosophical implications

22. Leah Dorn & Kenna Norder

The Power of the Power Series

This presentation breaks down the history, proof, and applications of power series. We begin by defining convergency and divergency of a power series, the components of the series, and the difference between a Taylor and a McLaurin series. We then go over the different techniques used to find the radius of convergence by using the root and ratio test. Following are several examples of how power series are used to express infinite polynomials and irrational numbers, as well as videos to show how the approximation changes as more terms are added. Finally, we explain how known power series can be used to find similar, unknown power series.

23. Conor Wellman (CC)

Generalized Rosette Harmonic Mappings

A harmonic mapping f is a complex valued univalent harmonic function defined on a region in the complex plane. Rosette harmonic mappings are generalizations of the polynomial harmonic mappings through modifying the canonical decomposition with hypergeometric 2F1 factors. We expand upon the Rosette Harmonic Mappings, which have analytic and coanalytic parts with exterior angles of $\pi/2$ at the nodes. By introducing a parameter q, we define Generalized Rosettes, which have analytic and coanalytic parts with exterior angles of qπ. For appropriate parameters, these Generalized Rosettes 'lift' through Weierstrauss Enneper equations to the Generalized Rosette Minimal Surfaces. At small values of q, these surfaces approximate the classical minimal surface known as Enneper's Surface, creating a link between this classical surface and the triply periodic Rosette Minimal Surface. As q increases, the angles at the nodes of the surfaces become increasingly sharp, and the surfaces eventually become unbounded. Moreover, their projections are no longer univalent harmonic functions. By introducing a rotation angle β, with $0 \leq \beta \leq 2\pi$, we find that seemingly unrelated Generalized Rosette Surfaces are in fact part of an associated family, suggesting they are conjugate surfaces.

24. Mustafa Sameen (CC)

Autoformalization with AI: Translating Natural Language Mathematics into Lean 4

In this presentation, I will explore the innovative intersection of artificial intelligence and formal mathematics through the development of an autoformalization system that translates natural language mathematical statements into verified Lean 4 code. Leveraging large language models, my approach iteratively generates formal theorem statements, subjects them to automated typechecking using Lean, and refines them based on feedback from Lean's error messages. This process not only demonstrates the potential for AI to bridge the gap between informal mathematical reasoning and formal proof verification but also highlights the challenges involved in accurately capturing mathematical nuance. The talk will include a live demonstration, showcasing how the system refines its output in real-time until a valid formalization is achieved. Attendees will gain insights into both the underlying technology and its implications for future research and education in mathematics.